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Proportional Navigation Through Predictive Control

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Introduction

A NEW formulation of the proportional navigation guidance (PNG) law using recently developed continuous-time predictive control approach is proposed. With certain assumptions, this guidance law exhibits similarity with some of the well-known versions of PNG laws. Simulations are carried out to assess the performance of this guidance law in comparison with conventional PNG against maneuvering targets. The results show that the performance of the present formulation is quite superior as compared to that of PNG.

Continuous-Time Predictive Control

The continuous-time predictive control approach offers an alternative strategy for designing controllers for nonlinear systems. In this approach, first introduced by Lu,[1] the state or output response of the nonlinear dynamic system is predicted by appropriate expansions and a quadratic cost function based on the predicted errors in the actual response, and the desired response and current control expenditure is minimized pointwise to obtain an optimal, nonlinear feedback control law. The performance index was modified by Singh et al.[2] for output tracking problems of input–output feedback linearizable systems, wherein the cost function is based on a function consisting of predicted errors, as well as their derivatives and integral. A brief outline of this approach is presented here for the sake of completeness. The reader is referred to Refs. 1 and 2 for more complete development.

Consider a single input/single output nonlinear system

$$\dot{x} = f(x) + g(x)u, \qquad y = c(x) \tag{1}$$

Received March 27, 1998; revision received July 13, 1998; accepted for publication July 14, 1998.

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assumed to be input–output feedback linearizable,[3] where $\boldsymbol{x} \in R^n$ is the state vector, y and u are the scalar output and input, $f(\boldsymbol{x})$ and $c(\boldsymbol{x})$ are continuously differentiable nonlinear functions, and $g(\boldsymbol{x})$ is a continuous function of $\boldsymbol{x}(t)$.

Suppose the desired output response of the system is given by a reference trajectory $y^*(t)$. The desired trajectory is such that it satisfies the system (1). Let the relative degree of y be γ. Then differentiating y by γ times yields

$$y^{(\gamma)} = a(\boldsymbol{x}) + \mathrm{D}(\boldsymbol{x})u \tag{2}$$

where $y^{(\gamma)}$ is γth derivative of y with respect to time. Now define a function s as

$$s \triangleq e^{(\gamma-1)} + k_{\gamma-1}e^{(\gamma-2)} + \cdots + k_1 e + k_0 x_s, \qquad \dot{x}_s = e \tag{3}$$

where $e(t) \triangleq y(t) - y^*(t)$ is the output tracking error and $k_i > 0$ are the gains to be chosen by the designer such that the polynomial

$$\lambda^\gamma + k_{\gamma-1}\lambda^{(\gamma-1)} + \cdots + k_1\lambda + k_0 \tag{4}$$

is Hurwitz. The function s is a linear combination of the tracking error e, its derivatives, and includes an integral term as well. Differentiating Eq. (3) gives

$$\dot{s} = y^{(\gamma)} - y^{*(\gamma)} + z \tag{5}$$

where $z \triangleq (k_{\gamma-1}e^{(\gamma-1)} + \cdots + k_1\dot{e} + k_0 e)$. To obtain the predictive controller, consider a performance index as

$$J(u) = \tfrac{1}{2}\{qs^2(t+h) + ru^2(t)\} \tag{6}$$

where $q > 0$ and $r \geq 0$ are weightings to be chosen, $h > 0$ is the prediction horizon, and $s(t+h)$ is the predicted value of s at the instant $(t+h)$. The predicted value of $s(t)$ at time $(t+h)$ is obtained by expanding $s(t+h)$ in a first-order Taylor series, and using Eqs. (2), (3), and (5) yields

$$s(t+h) \approx s(t) + h\left(a + z - y^{*(\gamma)} + \mathrm{D}u\right) \tag{7}$$

By applying the necessary condition for minimization of J with respect to u, i.e., $\partial J/\partial u = 0$, a closed-form solution for the control command $u(t)$ is obtained as

$$u = -\frac{h\,\mathrm{D}q}{(r + h^2\mathrm{D}^2 q)}\left[s + h\left(a + z - y^{*(\gamma)}\right)\right] \tag{8}$$

To obtain satisfactory tracking performance, it is necessary to choose proper values for the gains k_i, the weightings q and r, and the prediction horizon h. In Ref. 2, it has been shown that controller (8) with $r = 0$ achieves asymptotic tracking of the desired output trajectory and also offers robustness in presence of parametric uncertainty.

Formulation of Guidance Law

Consider a two-dimensional engagement geometry as shown in Fig. 1, where the missile and target are treated as point masses.

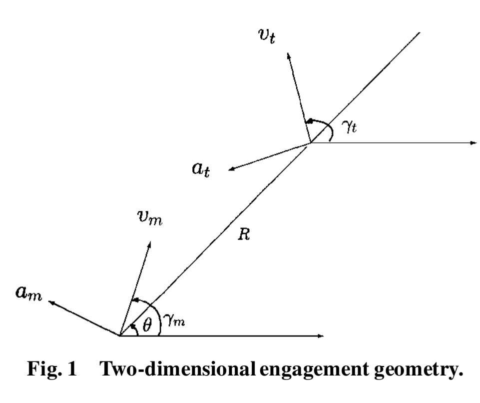

Fig. 1 Two-dimensional engagement geometry.

We assume that the velocity of missile and target remains constant throughout the engagement, with the missile velocity being greater than that of the target. Further, we neglect the autopilot and seeker loop dynamics. Under these assumptions, the engagement model can be represented by the following differential equations:

$$\dot{R} = v_t \cos(\gamma_t - \theta) - v_m \cos(\gamma_m - \theta)$$

$$\dot{\theta} = (1/R)[v_t \sin(\gamma_t - \theta) - v_m \sin(\gamma_m - \theta)] \tag{9}$$

$$\dot{\gamma}_t = a_t/v_t, \qquad \dot{\gamma}_m = a_m/v_m = a_c/v_m$$

where R is relative range from missile to target, θ is line of sight (LOS) angle, γ_m and γ_t are flight-path angles of the missile and the target, a_t is the lateral acceleration of the target, and a_m and a_c are the lateral and commanded accelerations of missile.

The objective of proportional navigation is to derive the LOS rate between the missile and its target to zero while closing on the target. To ensure this, we choose the output of system (9) as

$$y = c(\boldsymbol{x}) = (1/R)[v_t \sin(\gamma_t - \theta) - v_m \sin(\gamma_m - \theta)] = \dot{\theta} \tag{10}$$

Note that with LOS rate as output, the relative degree as stated in Eq. (2) is one, and the desired reference output trajectory is

$$y^*(t) = \dot{y}^*(t) = 0 \tag{11}$$

Following the procedure outlined in the earlier section, the various quantities appearing in Eq. (8) are

$$a(x) = -\frac{2\dot{R}\dot{\theta}}{R} + \frac{a_t \cos(\gamma_t - \theta)}{R}, \qquad D(x) = -\frac{\cos(\gamma_m - \theta)}{R}$$

$$s = \dot{\theta} + k_0 \int \dot{\theta}\,\mathrm{d}t, \qquad z = K_0\dot{\theta} \tag{12}$$

Substituting Eq. (12) in Eq. (8) yields the predictive PNG law as

$$a_c = \frac{h^2 q \cos(\gamma_m - \theta)}{\left[rR^2 + h^2 q \cos^2(\gamma_m - \theta)\right]} \times \left\{\frac{R\dot{\theta}}{h} + \frac{Rk_0\theta}{h} - 2\dot{R}\dot{\theta} + a_t \cos(\gamma_t - \theta) + Rk_0\dot{\theta}\right\} \tag{13}$$

where a_c is the commanded lateral acceleration of the missile. The guidance law (13), under certain assumptions, exhibits similarity with some of the known PNG versions. In Eq. (13), if we set $k_0 = 0$, the resulting guidance law is same as obtained from the output tracking control law discussed in Ref. 1, where the performance measure (6) consists of $e(t+h)$ in place of $s(t+h)$. Further, if one sets $r = 0$ in Eq. (13), in addition to $k_0 = 0$, one gets

$$a_c = \frac{1}{\cos(\gamma_m - \theta)}\left\{\frac{R\dot{\theta}}{h} - 2\dot{R}\dot{\theta} + a_t \cos(\gamma_t - \theta)\right\} \tag{14}$$

Bezick et al.[4] have presented the design of an input–output feedback linearizing homing guidance law based on the geometric control theory, where the relative range from missile to target R is chosen as output of the system (9). In present case with LOS rate as output, one can show that the guidance law (14) is a special case of the input–output feedback linearizing control law, for if one defines the external input $v = -(\dot{\theta}/h)$, substituting in Eq. (14), using Eqs. (12) and (2), and noting that the relative degree is one yield

$$\dot{y} = v \tag{15}$$

which is the linear relationship between output and input. Next, one can verify that the guidance law (14) closely resembles to the modified PNG (MPNG) law proposed by Ha et al.[5] Also the switched biased PNG, as derived in Ref. 6, shows similarity with Eq. (14) except that the target acceleration term is replaced by the switched bias term. In fact, the authors[6] have shown that the switched bias term represents an estimate of the target acceleration.

Simulations and Results

To assess the performance of the predictive PNG law and to compare it with that of PNG, we consider the target to be maneuvering with time-varying acceleration. It is important to note that for interception of the target, it is necessary to ensure $\dot{R} < 0$ while driving the LOS rate to zero. In Ref. 5, Ha et al. presented analytical conditions under which target interception is guaranteed for the conventional PNG and MPNG laws. In this work, because the performance comparison of the predictive guidance with that of PNG is studied and because the predictive guidance law, under certain assumptions, resembles the MPNG law, we consider the initial conditions of the missile–target engagement such that they satisfy the same analytical conditions derived in Ref. 5. Also, it is assumed that the angle between missile velocity vector and LOS remains acute throughout the engagement, i.e., $|\gamma_m - \theta| < 90$ deg.

The target acceleration profile is taken as $a_t = -80$ m/s^2 for $0 < t \leq 2$ s, $a_t = 0$ m/s^2 for $2 < t \leq 4$ s, and $a_t = 80$ m/s^2 for $t > 4$ s.

The other necessary simulation data, as taken from Ref. 6, are $R(0) = 4500$ m, $\theta(0) = 20$ deg, $\gamma_t(0) = 140$ deg, $\gamma_m(0) = 60$ deg, $v_m = 500$ m/s, and $v_t = 300$ m/s. The performance of the predictive proportional guidance law (13) is compared with the conventional PNG law.

Now rewriting Eq. (13) as

$$a_c(\text{pred}) = K\{-N'\dot{R}\dot{\theta} + (Rk_0\theta/h) + a_t \cos(\gamma_t - \theta)\} \tag{16}$$

where

$$K \triangleq \frac{h^2 q \cos(\gamma_m - \theta)}{rR^2 + h^2 q \cos^2(\gamma_m - \theta)}, \qquad N' \triangleq \frac{2\dot{R}h - R - hRk_0}{\dot{R}h} \tag{17}$$

where N' is the effective navigation ratio. For the purpose of valid comparison, the navigation ratio of the PNG law is modified by including the K term of Eq. (17). With this modification, the resulting PNG law is

$$a_c(\text{PN}) = -KN'\dot{R}\dot{\theta} \tag{18}$$

To carry out the comparison using a common basis, the navigation ratio N' is chosen as 4 in both the cases. However, as is evident from Eq. (17), to keep $N' = 4$ requires certain adjustments. To this end, we modulate the value of the prediction horizon h in Eq. (17), such that $N' = 4$ throughout the engagement. The resulting value of h is

$$h = \frac{R}{-2\dot{R} - k_0 R} > 0 \tag{19}$$

and the value of $k_0 > 0$ is chosen to ensure $h > 0$. The purpose of obtaining h through Eq. (19) is only to ensure that the effective navigation ratio is the same in both the cases for valid comparison. In fact, the performance of the predictive proportional guidance may be better for some other values of h and k_0 than the obtained from Eq. (19). The weightings in Eq. (17) are chosen as $r = 0.00001$ and $q = 1$. Figure 2 shows the commanded accelerations obtained from both guidance laws till $R \leq 5$ m, and one can observe that the acceleration profile due to predictive guidance is far more reasonable compared to that of PNG. The acceleration profile of the predictive guidance exhibits corners at time $t = 2$ and 4 s due to the step change in the target acceleration at these instants. Figure 3 shows the LOS rates for both laws, which clearly shows that the LOS rate with predictive guidance tends to zero near target interception, unlike that of PNG law.

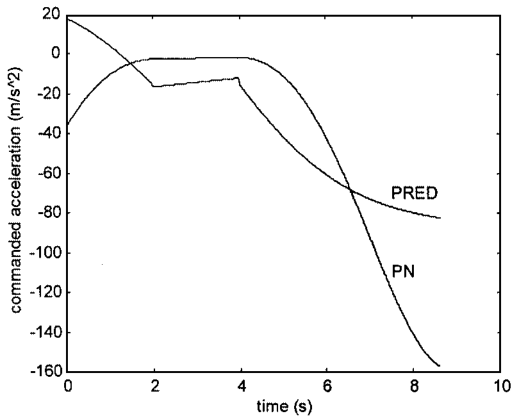

Fig. 2 Commanded acceleration profiles.

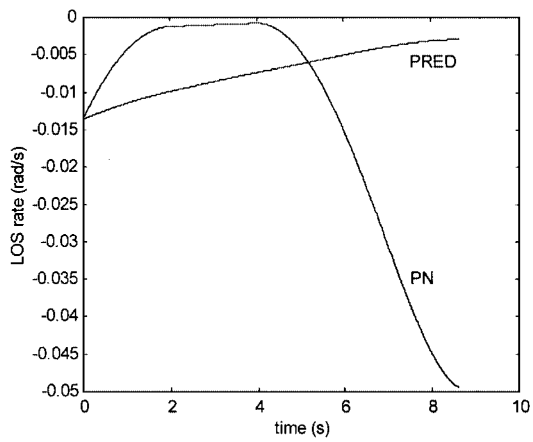

Fig. 3 LOS rate profiles.

Conclusion

In this Note, a new formulation of PNG is derived based on the recently developed continuous-time predictive control approach. Simulations have been carried out to assess the performance of this guidance law in comparison with conventional PNG for a maneuvering target, and results are presented. The results show that the present guidance law gives superior performance compared with the conventional PNG law.

Acknowledgments

The first author acknowledges S. B. Phadke, G. C. Pant, Chairman of the Faculty of Guided Missiles, and G. S. Mani, Director and Dean of the Institute of Armament Technology, for providing motivation and support during the course of this work.

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Guidance Strategy for Solid Propelled Launchers

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Introduction

THIS Note describes the guidance strategy used for the Brazilian satellite launcher,[1] called VLS. The vehicle has four stages and uses solid fuel; the last stage is spin stabilized. Its first mission was to place a 115-kg satellite into a 750-km altitude circular orbit. Because there is no velocity control—the burning of the fourth stage cannot be interrupted—the final orbit is determined by the suborbital trajectory (ballistic phase) between the third and fourth stages, the ignition time of the fourth stage, and its inertial attitude. Thus, to achieve the prescribed altitude of a circular orbit, the last stage must be in a particular ballistic trajectory and the last action of the control system is to point the fourth-stage/satellite assembly to a certain inertial direction and to decide the ignition time.

The guidance commands produce, during the third stage, the reference to the attitude control system so that the vehicle reaches a transfer orbit with prespecified parameters, called parametric suborbit guidance (PSG). Regarding the fourth stage, the guidance command sets the inertial direction and establishes the ignition time. This is called the pointing algorithm (PA).

Pointing Algorithm

The PA[2,3] calculates the ignition time and the inertial attitude for the fourth stage based on real navigation data to obtain a proper transfer to a circular orbit. The strategy should operate during the coast phase between the third-stage burnout and the fourth-stage ignition. The strategy proposed is similar to an impulsive orbit transfer because the fourth-stage energy is considered as an increment of velocity. The main difference to the impulsive assumption is the ignition time. The real ignition time has an additional term that is obtained based on conservation of energy. After this, the obtained deployment attitude[4] is related to the inertial frame. The algorithm assumes that no disturbance is present during the coast phase to change that Keplerian orbit.

During the coast phase and the burning of the last stage, the equation of motion of the vehicle center of mass[4] is $\ddot{\boldsymbol{R}} = g(\boldsymbol{R}) + \Gamma(t)\cdot\boldsymbol{\vartheta}$, where $\boldsymbol{R}$ is the vehicle radius vector, g is the gravitational acceleration, $\boldsymbol{\vartheta}$ is a constant unit vector in the thrust direction (kept constant by the attitude control system), and $\Gamma(t)$ is the propulsive acceleration that is assumed to be a well-known function. The solution of $\boldsymbol{R}(t)$ can be formulated as

$$\boldsymbol{V} = \boldsymbol{V}_0 + \Delta V\cdot\vartheta + \Delta\boldsymbol{V}_g$$

$$R = R_0 + (t - t_0)\cdot\boldsymbol{V}_0 + \Delta R\cdot\vartheta + \Delta\boldsymbol{R}_g$$

where

$$\Delta\boldsymbol{V}_g = \int_{t_0}^{t}\frac{\mu\boldsymbol{R}(\xi)}{R(\xi)^3}\,\mathrm{d}\xi, \qquad \Delta V = \int_{t_{ig}}^{t}\Gamma(\xi)\,\mathrm{d}\xi$$

t_{ig} is the ignition time with $t_0 \le t_{ig}$, and ΔR and $\Delta\boldsymbol{R}_g$ are, respectively, the integration of ΔV and $\Delta\boldsymbol{V}_g$.

The increment of angular momentum over the last stage is given by $\Delta\boldsymbol{H} = \boldsymbol{R}\times m\cdot\boldsymbol{V} - \boldsymbol{R}_0\times m_0\cdot\boldsymbol{V}_0$. The motion is considered to

Received Dec. 29, 1997; revision received July 13, 1998; accepted for publication July 20, 1998.

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